Conditional Peak Ground Accelerations in the Canterbury Earthquakes for Conventional Liquefaction Assessment

Technical Report Prepared for the Department of Building and Housing

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Executive Summary

The 4 September 2010, 22 February 2011 and 13 June 2011 earthquakes caused strong ground motions in the densely populated Christchurch and surrounding Canterbury region, resulted in the largest earthquake damage in New Zealand's history since European settlement. Among other things, these ground motions caused severe liquefaction of surficial soils over large regions resulting in extensive damage to lifelines, residential, commercial and industrial structures.

A critical step in the seismic performance assessment of existing facilities is to examine observed performance against performance predicted using conventional assessment methodologies. For liquefaction evaluation a critical input in such assessments is the level of peak ground acceleration (PGA) that structures were subjected to in the series of earthquake events.

This document provides an overview of the development of conditional PGA values observed on alluvial deposits in the greater Christchurch region in the 4 September 2010, 22 February 2011 and 13 June 2011 earthquakes. The predicted values are dependent on both the general manner in which PGAs are observed to vary over a region from a given causative fault (as predicted by empirical ground motion models), combined with the actual recorded PGA values at various strong motion stations in the region. As such, the predicted PGA values are termed 'conditional', that is, the prediction is conditional on the observations at distinct locations.

The conditional prediction of PGA at a given location from a given earthquake event is not a single deterministic number, but in the form of a probability distribution with median and standard deviation. Where the ground motion is known exactly (i.e. at strong motion stations), the uncertainty (i.e. standard deviation) in the prediction is zero. However, for general locations uncertainty exists. It is strongly recommended that this uncertainty in PGA is considered by those tasked with liquefaction assessments.

1. Introduction

This document provides an overview of the development of conditional PGA values occurring on alluvial deposits in the greater Christchurch region in the 4 September 2010, 22 February 2011 and 13 June 2011 earthquakes. Section 2 provides background on conventional liquefaction evaluation, and hence the need for ascertaining the predicted PGA values at general locations during the aforementioned earthquake events. Section 3 provides the computed values of conditional PGA, as well as the underlying theoretical details upon which the calculations are based. Section 4 discusses the use of the predicted conditional PGA distribution for liquefaction assessments in past events, in particular the role of the magnitude scaling factor (MSF), and consideration of uncertainty in the prediction of PGA. Section 5 summarises the key points in this document.

2. Simplified method for liquefaction evaluation

Liquefaction assessments conventionally utilise a stress-based approach in which the factor of safety (FS) against liquefaction is obtained from the cyclic stress ratio (*CSR*) and the cyclic resistance ratio (*CRR*) (New Zealand Geotechnical Society 2010). Specifically,

$$FS = \frac{CRR_{7.5}}{CSR_{7.5}} \tag{1}$$

where the subscripts in both the denominator and numerator indicate that the ratios are representative of a $M_w 7.5$ earthquake. The *CRR* can be obtained via various insitu testing methods (e.g. CPT, SPT, Vs) or laboratory data, but importantly is a property of the geotechnical conditions at the site of concern (New Zealand Geotechnical Society 2010). The *CSR*, which represents the ratio between the cyclic shear stress and vertical effective stress in the soil, can be estimated using the general equation:

$$CSR_{7.5} = 0.65 \frac{a_{max}}{g} \frac{\sigma_{vo}}{\sigma'_{vo}} r_d \frac{1}{MSF} \tag{2}$$

where a_{max} is the average horizontal (geometric mean) peak ground acceleration (PGA) at the ground surface; g is the acceleration of gravity; σ_{vo} is the vertical total stress at the depth of interest; σ'_{vo} is the vertical effective stress at the depth of interest; r_d is a reduction factor to account for the soil flexibility; and MSF is a magnitude scaling factor to account for the number of cycles of significant ground motion.

The estimation ofa_{max} which is the peak ground acceleration in units of gravity, for the observed earthquakes in the Canterbury region is the principal subject of this document and is elaborated upon in section 3. Section 4 discusses the application of the results of section 3 in liquefaction assessments, including the consideration of the magnitude scaling factor, *MSF*.

3. Conditional PGA distributions from the Canterbury earthquakes

3.1. Theory for conditional PGA calculations

Because of the complexity of a ground motion time series, the engineering representation of ground motion severity typically comprises one or more ground motion intensity measures. Here only the intensity measure of peak ground acceleration (PGA) is considered, although the theory below is applicable to any other intensity measure.

The representation of PGA at a single location i, for the purposes of ground motion prediction, is generally given by:

$$lnPGA_i = ln\overline{PGA_i}(Site, Rup) + \eta + \epsilon_i \tag{3}$$

where $lnPGA_i$ is the (natural) logarithm of the observed PGA; $ln\overline{PGA_i}(Site, Rup)$ is the median of the predicted logarithm of PGA as given by an empirical ground motion prediction equation (GMPE), which is a function of the site and earthquake rupture considered; η is the inter-event residual; and ϵ_i is the intra-event residual. Based on equation (3), empirical ground motion prediction equations can provide the (unconditional) distribution of ground motion shaking as:

$$lnPGA_i \sim N(ln\overline{PGA_i}, \sigma_\eta^2 + \sigma_\epsilon^2) \tag{4}$$

where $X \sim N(\mu_X, \sigma_X^2)$ is short-hand notation for X having a normal distribution with mean μ_X and variance σ_X^2.

By definition, for a given ground motion intensity measure, (e.g. peak ground acceleration, *PGA*) all observations from a single earthquake event have the same inter-event residual, η. In this regard, the inter-event residual represents the correlation between all observations from a single event, which may occur as a result of a unique effect occurring during the earthquake rupture, which subsequently affects the ground motion at all locations in a systematic manner. On the other hand, the intra-event residual, ϵ_i varies from site to site. In this regard the intra-event residual represents all other randomness which leads to a difference between the observed ground motion intensity, the predicted median ground motion intensity, and the systematic inter-event residual. While the intra-event residual varies from site to site, it is correlated spatially as a result of similarities of path and site effects between various locations.

Based on the aforementioned properties of η and ϵ_i, use can be made of recorded PGA values at strong motion stations to compute a conditional distribution of PGA at an arbitrary site of interest. The required steps are discussed below.

Firstly, an empirical ground motion prediction equation (GMPE) is used to compute the unconditional distribution of ground motion intensity at the strong motion stations where ground motions were recorded. A mixed-effects regression (Abrahamson and Youngs 1992, Pinheiro et al. 2008) can then be used to determine the inter-event residual, η, and the intra-event residuals, ϵ_i's, for each strong motion station.

Secondly, the covariance matrix of intra-event residuals is computed by accounting for the spatial correlation between all of the strong motion stations and the site of interest. The joint distribution of intra-event residuals at the site of interest and the considered strong motion stations can be represented by:

$$\begin{bmatrix} \epsilon^{site} \\ \boldsymbol{\epsilon^{SMstation}} \end{bmatrix} = N\left(\begin{bmatrix} 0 \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \sigma^2_{\epsilon_{site}} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{bmatrix}\right) \tag{5}$$

where $\boldsymbol{X} \sim N(\boldsymbol{\mu_X}, \mathbf{\Sigma})$ is short-hand notation for $\boldsymbol{X}$ having a multivariate normal distribution with mean $\boldsymbol{\mu_X}$ and covariance matrix $\mathbf{\Sigma}$ (i.e. as before, but in vector form); and $\sigma^2_{\epsilon_{SMstation}}$ is the variance in the intra-event residual. In Equation (5) the covariance matrix has been expressed in a partitioned fashion to elucidate the subsequent computation of the conditional distribution of ϵ^{site}. The individual elements of the covariance matrix can be computed from:

$$\mathbf{\Sigma}(i,j) = \rho_{i,j}\sigma_{\epsilon_i}\sigma_{\epsilon_j} \tag{6}$$

where $\rho_{i,j}$ is the spatial correlation of intra-event residuals between the two locations i and j; and σ_{ϵ_i} and σ_{ϵ_j} are the standard deviations of the intra-event residual at locations i and j. Based on the joint distribution of intra-event residuals given by Equation (5) the conditional distribution of ϵ^{site} can be computed from (Johnson and Wichern 2007):

$$\begin{aligned} \left[\epsilon^{site} \middle| \boldsymbol{\epsilon^{SMstation}}\right] &= N\left(\mathbf{\Sigma}_{12} \cdot \mathbf{\Sigma}_{22}^{-1} \cdot \boldsymbol{\epsilon^{SMstation}}, \sigma^2_{\epsilon_{site}} - \mathbf{\Sigma}_{12} \cdot \mathbf{\Sigma}_{22}^{-1} \cdot \mathbf{\Sigma}_{21}\right) \\ &= N\left(\mu_{\epsilon^{site}|\boldsymbol{\epsilon^{SMstation}}}, \sigma^2_{\epsilon^{site}|\boldsymbol{\epsilon^{SMstation}}}\right) \end{aligned} \tag{7}$$

Thirdly, using the conditional distribution of the intra-event residual at the site of interest given by Equation (7) and substituting into Equation (4), the conditional distribution of peak ground acceleration at the site of interest, PGA_{site} can be computed from:

$$\begin{aligned} &[lnPGA_{site}|ln\boldsymbol{PGA_{SMstation}}] \\ &\qquad = N\left(ln\overline{PGA_{site}} + \eta + \mu_{\epsilon^{site}|\boldsymbol{\epsilon^{SMstation}}}, \sigma^2_{\epsilon^{site}|\boldsymbol{\epsilon^{SMstation}}}\right) \end{aligned} \tag{8}$$

That is, the conditional distribution of PGA at a specific site is a lognormal random variable (i.e. the log of PGA is a normal random variable) which is completely defined via the conditional median and conditional standard deviation. Thus, the results presented in section 3.4 provide these conditional median and conditional standard deviation values. Section 4 discusses how these two values can be used in liquefaction assessments.

It should be noted that in cases where the site of interest is located far from any strong motion stations the conditional distribution will be similar to the unconditional distribution, and for sites of interest located very close to a strong motion station the conditional distribution will approach the value observed at the strong motion station.

In the following analyses the NZ-specific GMPE developed by Bradley, and the spatial correlation model Goda and Hong (2008) were adopted. The applicability of the Bradley (2010) GMPE for PGA in the Christchurch earthquakes, in particular, is explicitly examined in the next section.

3.2. Peak ground accelerations observed in the Canterbury earthquakes and comparison with empirical predictions

As shown in the previous section, the conditional prediction of PGA is influenced by the unconditional prediction of PGA obtained from ground motion prediction equations.

Table 1 provides the numerical values of PGA observed at strong motion stations during the Canterbury earthquakes (Bradley 2012a, Bradley and Cubrinovski 2011). A large number of ground motions were observed in these events. This wealth of recorded data helps to provide significant constraint to the predicted PGA values over the entire Canterbury region.

Table 1: Observed values of PGA at strong motion stations during the 4 September 2010, 22 February 2011, and 13 June 2011 (2:20pm) earthquakes.

Event	4 September 2010 ($M_w 7.1$)	22 February 2011 ($M_w 6.2$)	13 June 2011 ($M_w 6.0$)
Station			
CACS	0.197	0.211	0.136
CBGS	0.158	0.501	0.163
CCCC	0.224	0.429	-
CHHC	0.173	0.366	0.215
CMHS	0.237	0.370	0.178
HPSC	0.147	0.216	0.256
HVSC	0.606	1.412	0.914
KPOC	0.332	0.206	0.099
LINC	0.437	0.118	0.065
LPCC	0.290	0.916	0.639
NBLC	-	-	0.214
NNBS	0.206	0.674	0.198
PPHS	0.221	0.213	0.122
PRPC	0.214	0.628	0.341
REHS	0.252	0.522	0.264
RHSC	0.210	0.275	0.194
ROLC	0.340	0.184	0.045
SHLC	0.175	0.334	0.185
SMTC	0.176	0.161	0.085
TPLC	0.266	0.114	0.065

Figure 1-Figure 3 compare the observed and predicted PGA values for the three different earthquakes of concern. For the 4 September 2010 and 22 February 2011 earthquake the observed PGA values are largely consistent with the empirical prediction of Bradley (2010). While these two events are, on average, consistent with the empirical prediction there is significant scatter in the observations as a result of complex source, path and site effects. Because the conditional PGA values computed subsequently account for the spatial correlation in intra-event residuals, and hence the site-to-site variability, then they attempt to account for this localized variability in PGA values.

In contrast to the good comparison between the 4 September 2010 and 22 February 2011 earthquake observations and the Bradley (2010) GMPE, observations from the 13 June 2011 earthquake produced observed PGA values which are systematically lower than that predicted by the Bradley (2010) GMPE. The over-prediction is approximately 1 standard deviation (i.e. $\eta = 1.178$). As the conditional PGA values computed subsequently account for the inter-event residual (i.e. η), they will explicitly consider these lower-than-expected

PGA values, which indicates the possibility of a unique source effect which is present in this earthquake.

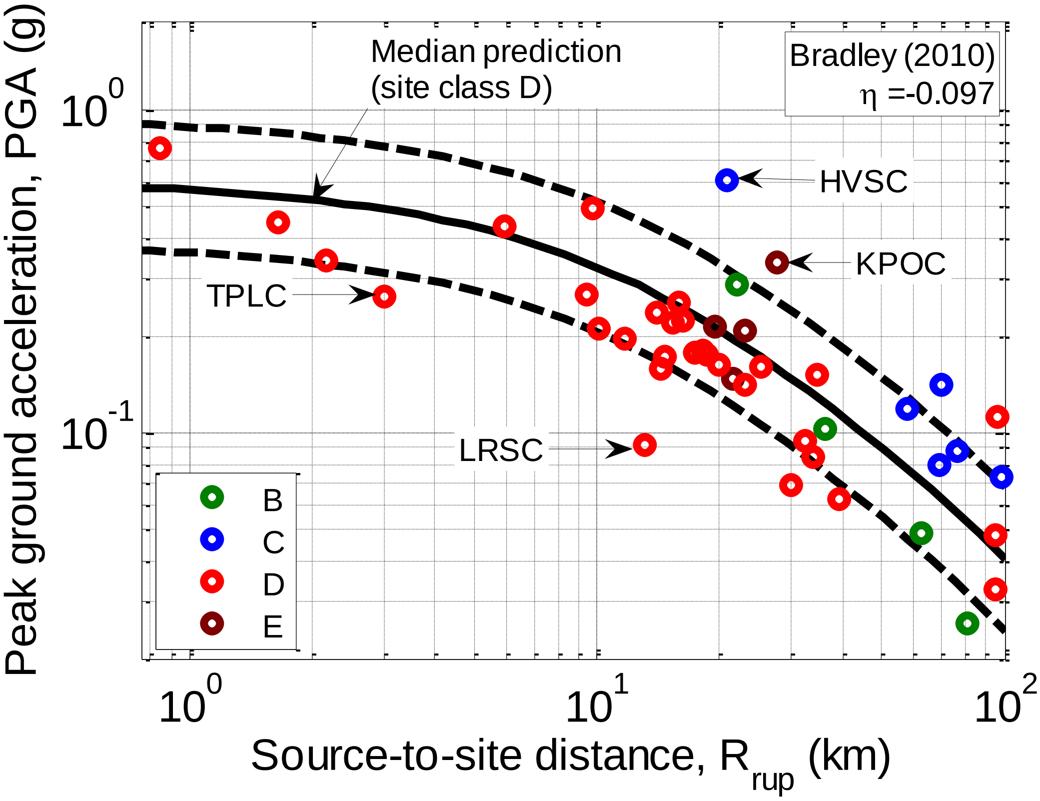


Figure 1: Comparison of observed PGA values with the empirical prediction of Bradley (2010) for the 4 September 2010 earthquake (M_w*7.1).*

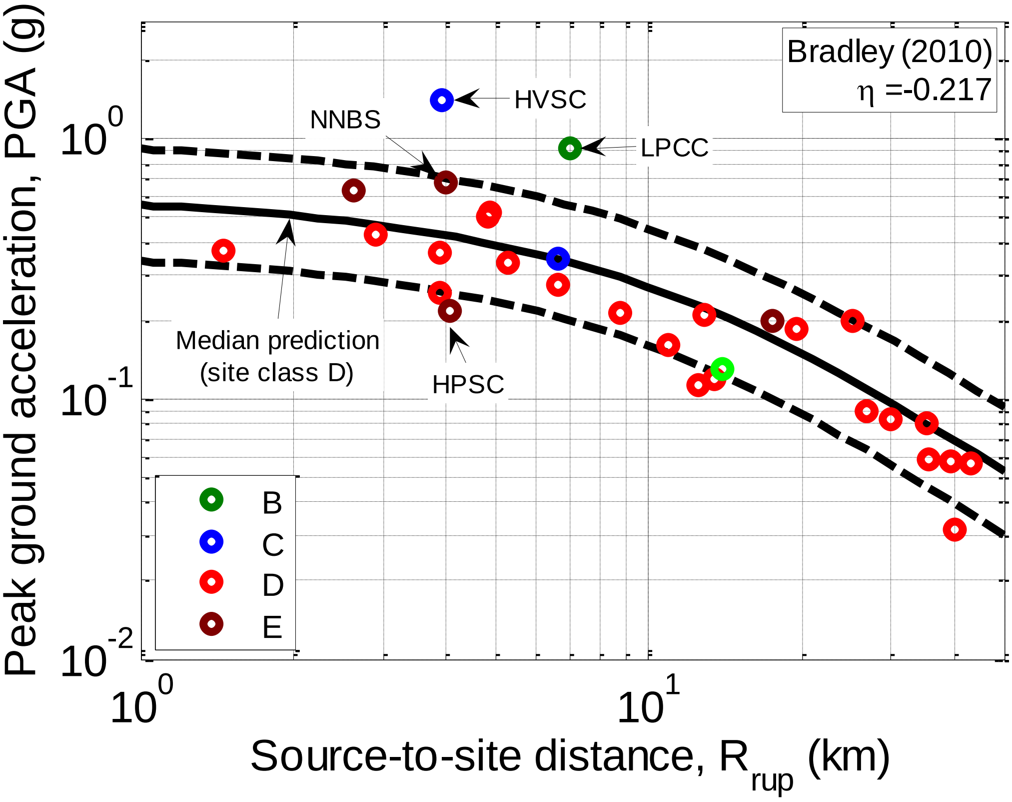


Figure 2: Comparison of observed PGA values with the empirical prediction of Bradley (2010) for the 22 February 2011 earthquake (M_w6.2).

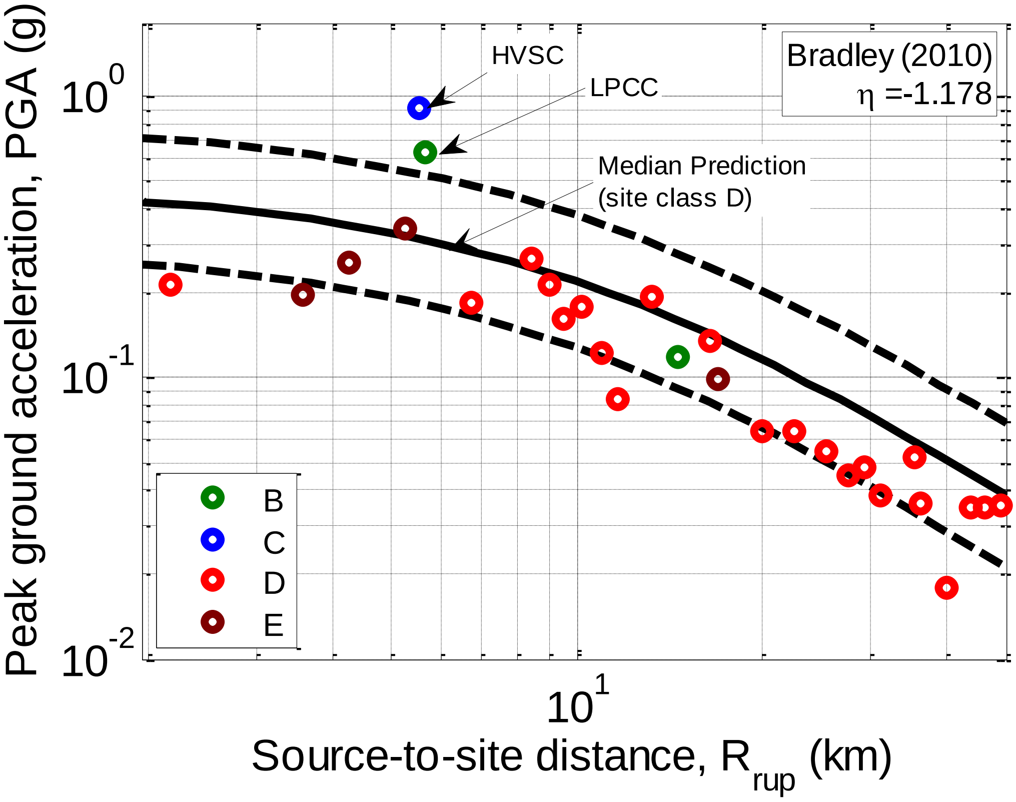


Figure 3: Comparison of observed PGA values with the empirical prediction of Bradley (2010) for the 13 June 2011 (2:20pm) earthquake (M_w6.0).

3.3. Spatial correlation of intra-event residuals

As previously noted, while the inter-event residual, η, for a given earthquake event is constant, the intra-event residual, ϵ_i, varies from observation to observation at different sites. Empirical analysis of ground motion data worldwide, however, illustrate the intra-event residuals are spatially correlated. This is inferred physically as the result of similarity in path and site effects at observation locations at geographically close distances. The correlation of ground motion amplitudes (e.g. PGA and spectral accelerations, SA) is vibration period-dependent since the natural frequency of waves is proportional to the length of the waves, and therefore the spatial distances over which waves are expected to be relatively coherent.

Figure 4 illustrates the spatial correlation model for PGA as a function of the separation distance between two locations. As expected on physical grounds, the correlation is 1.0 for a separation distance of zero (i.e. two points at the same location), and tends toward zero as the separation distance increases. Thus, on the basis of Figure 4 and the theoretical discussions in section 3.1, it can be understood that if the ground motion PGA is above that expected at a given strong motion station then it is more likely that the PGAs near this station will also be above average. The strength of this statement will decrease as the separation distance from the station and the site of interest increases.

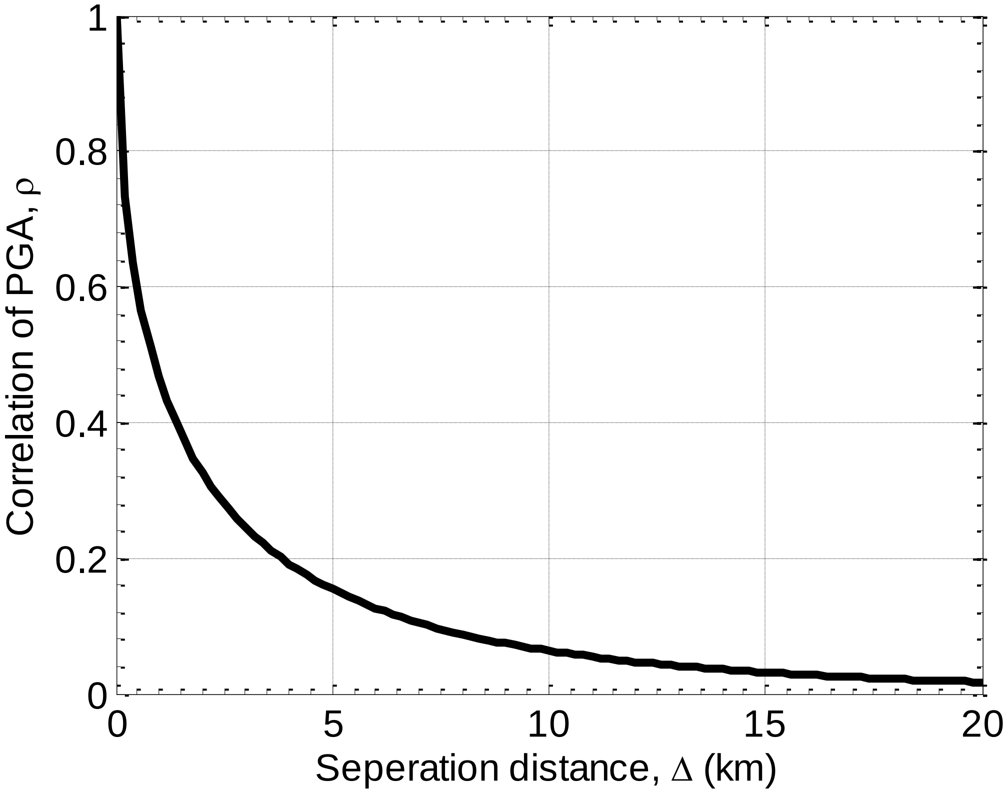


Figure 4: Correlation of intra-event resduals for PGA as a function of seperation distance (Goda and Hong 2008).

3.4. Computed PGA distributions in the Canterbury earthquakes

Figure 5-Figure 7 present the conditional median and conditional standard deviation of PGA from the 4 September 2010, 22 February 2011, and 13 June 2011 (2:20pm) earthquakes, respectively, in the form of contour maps. As noted in section 3.1, the conditional distribution of PGA is a lognormal random variable that can be defined via the conditional median and conditional standard deviation. Section 4 provides additional guidance on the use of these conditional values for liquefaction assessments.

Several features are worthy of note in the Figure 5-Figure 7:

- The median PGA amplitudes display a typical attenuation in amplitude as the distance from the earthquake source increases.
- In the proximity of strong ground motion stations, the contours can be observed to vary markedly as a result of differences between some observed PGA. This is consistently the case in Heathcote Valley for all events, due to strong basin-edge effects (Bradley 2012a, Bradley 2012b); and also apparent at Kaiapoi High School during the 4 September 2010 earthquake as a result of wave-guide effects (Bradley 2012a). However, as shown by the median PGA contours these effects are expected to be localised.
- The conditional standard deviations shown at the bottom panel of each of the figures provide an indication of the level of uncertainty in the conditional median PGA prediction. Near strong motion stations the conditional standard deviations decrease toward zero. This implies that the prediction of PGA is more accurate close to strong motion stations, and less accurate as the distance from strong motion stations increases.

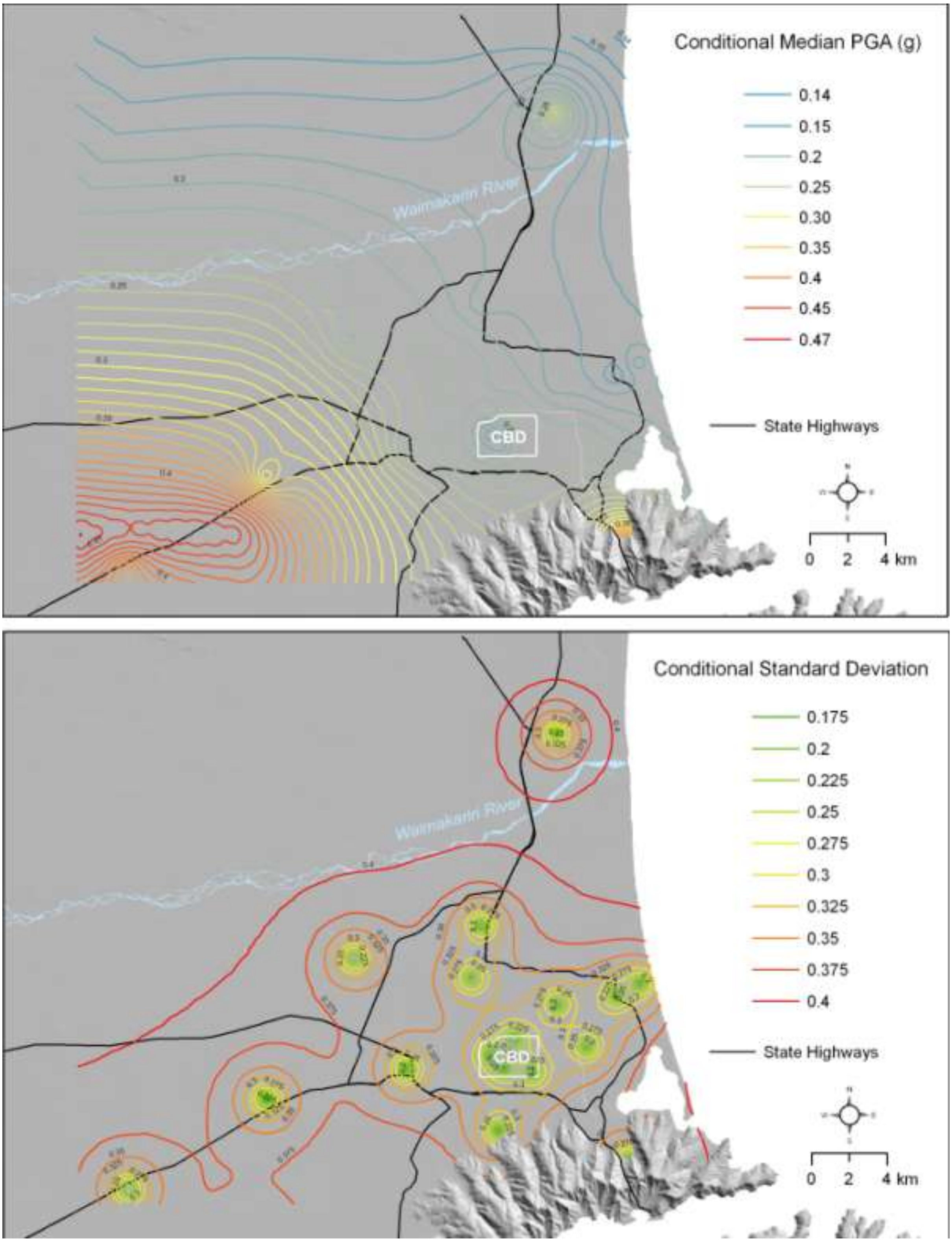


Figure 5: Conditional median (top) and conditional standard deviation (bottom) of PGA predicted in Canterbury from the 4 September 2010 earthquake.

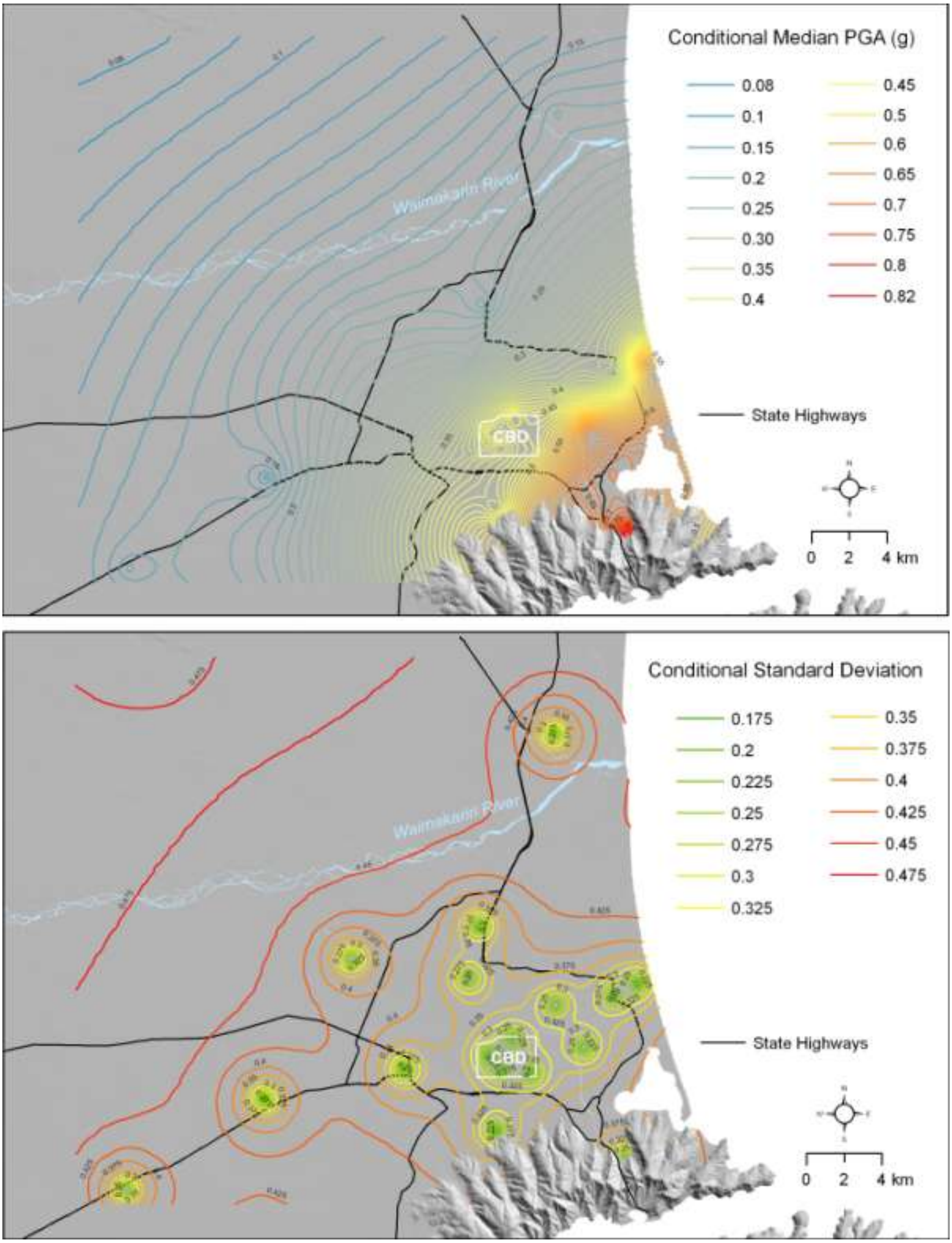


Figure 6: Conditional median (top) and conditional standard deviation (bottom) of PGA predicted in Canterbury from the 22 February 2011 earthquake.

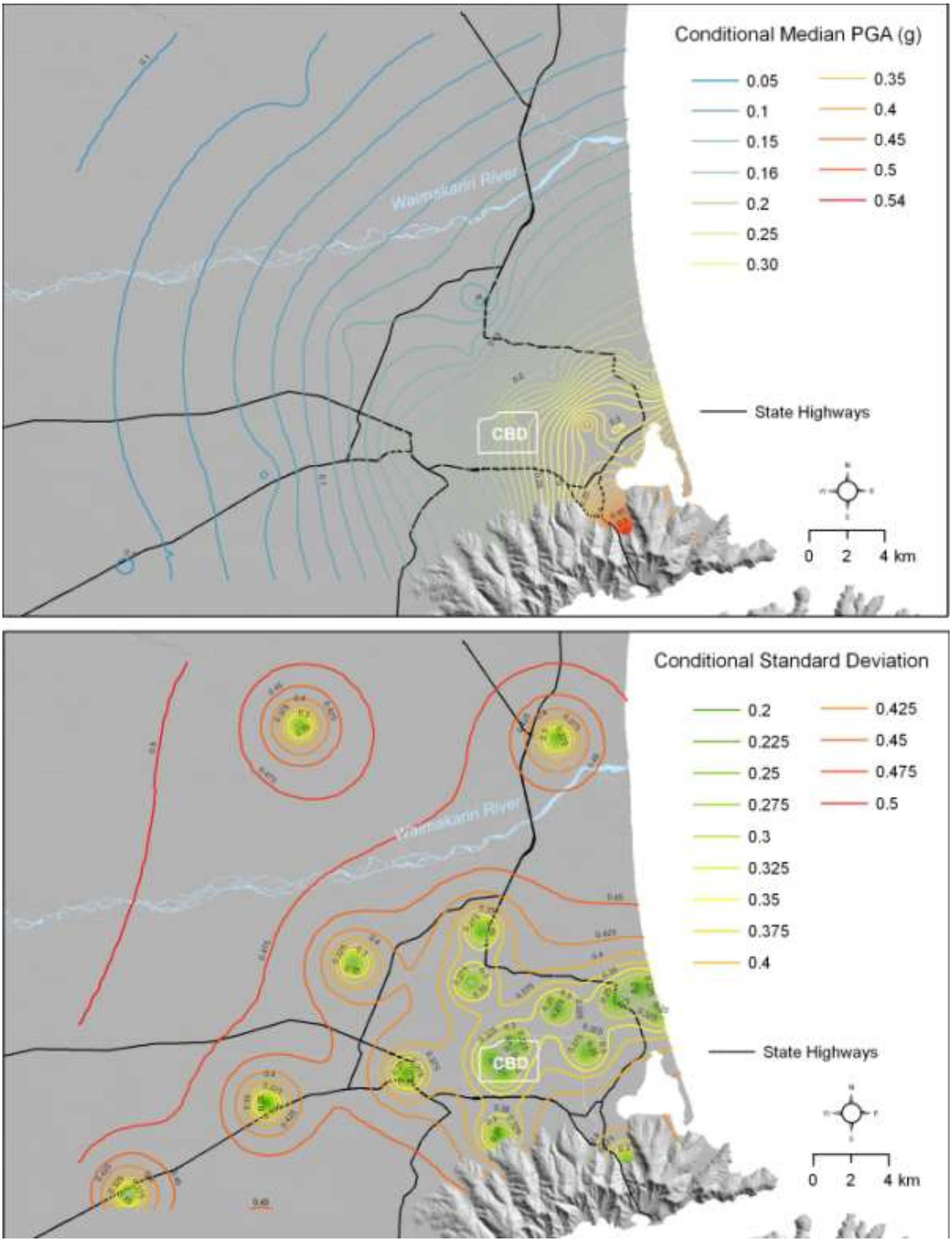
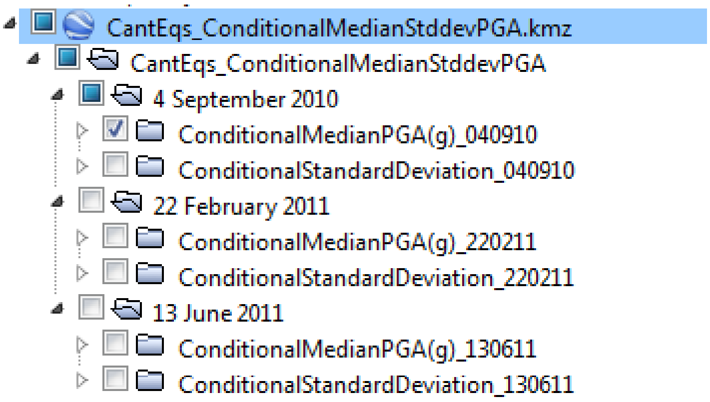


Figure 7: Conditional median (top) and conditional standard deviation (bottom) of PGA predicted in Canterbury from the 13 June 2011 earthquake.

3.5. Google earth files of PGA contours

In order to make use of the contour plots in Figure 5-Figure 7 for a site-specific liquefaction assessment, Google Earth files have been created and are appended with this report. The Google Earth .kmz file “CantEqs_ConditionalMedianStddevPGA” contains all the contour information. Figure 8 illustrates the folder hierarchy of this .kmz file. There is one folder for each of the three earthquake events and one sub-folder for the conditional median and conditional standard deviation.



- ■ CantEqs_ConditionalMedianStddevPGA.kmz
  - ■ CantEqs_ConditionalMedianStddevPGA
    - ■ 4 September 2010
      - ☑ ConditionalMedianPGA(g)_040910
      - ☐ ConditionalStandardDeviation_040910
    - ☐ 22 February 2011
      - ☐ ConditionalMedianPGA(g)_220211
      - ☐ ConditionalStandardDeviation_220211
    - ☐ 13 June 2011
      - ☐ ConditionalMedianPGA(g)_130611
      - ☐ ConditionalStandardDeviation_130611

Figure 8: Folder heirarchy of the Google Earth .kmz file.

The specific contour values can be ascertained by clicking on the contour as illustrated in Figure 9. For assessment of the specific values of conditional median PGA and conditional standard deviation PGA at a given location, visual interpolation of contour values can be used, since contour intervals are 0.01g for the conditional median PGA and 0.05 for the conditional standard deviation.

0.32

0.32

FID 24

CONTOUR 0.32

Figure 9: Ascertaining the value of a specific contour. For the case shown the selected contour has a value of median PGA = 0.32g.

4. Application of conditional PGA values for liquefaction assessment

4.1. $PGA_{7.5}$ values for a M_w7.5 event

The conditional PGA values presented in the previous section represent the conditional distribution of actual PGA observed in the three earthquakes. For liquefaction assessment, however, both the PGA and the effective number of cycles, as represented by the magnitude scaling factor, MSF, are needed. This was illustrated by Equation (2) in section 2.

Current best practice is that the magnitude scaling factor used for liquefaction triggering analysis should be consistent with the scaling factor used in the development of the triggering criteria being applied. The Department of Building & Housing residential foundation guidance (DBH 2011) specifies that for application of the guidance document, assessment of liquefaction triggering and its effects should be carried out in accordance with the method outlined in Idriss and Boulanger (2008). There are several additional analysis requirements, and the DBH guidance may change over time, so the most up-to-date version of the guidance document should be consulted to confirm the recommended procedures. For liquefaction assessment in accordance with Idriss and Boulanger, the following magnitude scaling factor is specified:

$$MSF = 6.9 * exp\left(-\frac{M_w}{4}\right) - 0.058 \leq 1.8 \tag{9}$$

where M_w is the moment magnitude of the earthquake event. . Table 2 provides the moment magnitudes of the three earthquakes and the computed values of MSF using Equation (9). It should be noted that other magnitude scaling factor expressions also exist, some of which contain limits on the maximum MSF value (e.g. Idriss and Boulanger 2008).

Table 2: Observed values of PGA at strong motion stations during the 4 September 2010, 22 February 2011, and 13 June 2011 (2:20pm) earthquakes. The values of MSF and $PGA_{7.5}/PGA$ are based on the use of Equation (9) and will vary if a different magnitude scaling relationship is used.

Event	Magnitude, M_w	MSF	$PGA_{7.5}/PGA$
4 September 2010	7.1	1.11	0.90
22 February 2011	6.2	1.41	0.71
13 June 2011	6.0	1.48	0.68

The term $\frac{a_{max}}{g}$ in Equation (2), which is the normalized PGA, can be multiplied by the reciprocal of the MSF to obtain the equivalent PGA for a M_w7.5 event, that is:

$$PGA_{7.5} = PGA\frac{1}{MSF} \tag{10}$$

Hence, the values of PGA observed in Figure 5 for the 4 September 2010 earthquake should be multiplied by a factor of 0.90 to obtained that value of $PGA_{7.5}$. This value is then used to obtain the $CSR_{7.5}$ from a modified form of Equation (2):

$$CSR_{7.5} = 0.65 PGA_{7.5} \frac{\sigma_{vo}}{\sigma'_{vo}} r_d \tag{11}$$

4.2. Consideration of uncertainty in the conditional PGA

As previously emphasised, the predicted values of PGA from these earthquakes are not known exactly (except at strong motion stations). The methodology for prediction of the conditional PGA values provides a distribution with both median and standard deviation. It is suggested that the uncertainty in the estimated PGA be explicitly considered in liquefaction assessments of existing structures in order to ensure that misinterpretations are not made when attempting to reconcile observed and predicted seismic performance.

The conditional distribution of PGA computed for the three events is a lognormal distribution. The conditional median PGA value represents the 50th percentile of this distribution, i.e. there is a 50% probability that the true PGA (which is unknown) is greater than this predicted median. In order to obtain another percentile of the PGA distribution, the following equation for the lognormal distribution can be used:

$$PGA_x = PGA_{50} * \exp\left(z_x \sigma_{lnPGA}\right) \tag{12}$$

where PGA_x is the value of PGA for the xth percentile; PGA_{50} is the 50th percentile (i.e. median) value of PGA; $\sigma_{lnPGA}$is the standard deviation of PGA (i.e. the bottom panel in Figure 5-Figure 7) and z_x is the 'z-value' of the standard normal distribution for the xth percentile which is the number of standard deviations from the median. In order to illustrate this consider the example below:

<u>Example:</u>

A liquefaction assessment is performed for a site in the Christchurch CBD to reconcile the observations in the 4th September 2010 earthquake. For the site location, interpolation of the provided contours in Figure 5 reveals that the conditional PGA was 0.2 g and the conditional standard deviation was 0.25.

Using Equation (12) the complete distribution of PGA can be computed as shown in Figure 10. The 16th and 84th percentiles, in particular represent one standard deviation either side of the median and hence can be computed from:

$$\begin{aligned} PGA_{16} &= PGA_{50} * \exp\left(-\sigma_{lnPGA}\right) \\ PGA_{84} &= PGA_{50} * \exp\left(+\sigma_{lnPGA}\right) \end{aligned} \tag{13}$$

It can be seen that the 16th and 84th percentile values of the conditional PGA distribution are 0.156 g and 0.257 g, respectively. Hence, one can state with a confidence of 68% (i.e. 84-16) that the actual PGA value that occurred was between 0.156 g and 0.257 g.

These different percentiles can be used directly in liquefaction assessments. For example, the CSR for a given percentile PGA can be obtained from:

$$CSR_{7.5(x)} = 0.65 PGA_{7.5(x)} \frac{\sigma_{vo}}{\sigma'_{vo}} r_d \tag{14}$$

Where (x) is used to represent the considered xth percentile. Similarly, the xth percentile value of the factor of safety against liquefaction can be obtained from:

$$FS_{(x)} = \frac{CRR_{7.5}}{CSR_{7.5(x)}} \tag{15}$$

Thus, one can form a distribution of the factor of safety against liquefaction. If there is significant uncertainty in the value of $CRR_{7.5}$ then this can be considered appropriately also.

As can be seen from this example, standard deviations of 0.25 are quite significant and hence there is potential for erroneous reconciliations of observed and predicted performance if only a single value of PGA is considered.

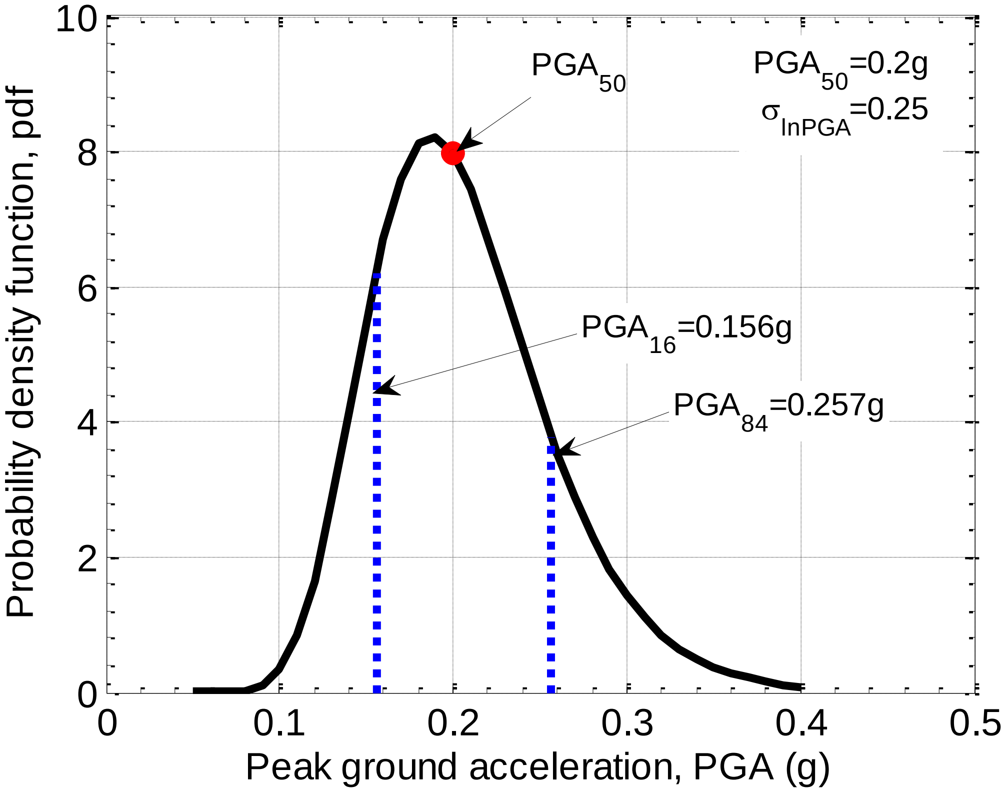


Figure 10: Example conditional distribution of PGA.

5. Conclusions

This document provides an overview of the development of conditional PGA values observed on alluvial deposits in the greater Christchurch region in the 4 September 2010, 22 February 2011 and 13 June 2011 earthquakes. The predicted values are dependent on both the general manner in which PGAs are observed to vary over a region from a given causative fault (as predicted by empirical ground motion models), combined with the actual recorded PGA values at various strong motion stations in the region. As such, the predicted PGA values are termed ‘conditional’, that is, the prediction is conditional on the observations at distinct locations.

The conditional prediction of PGA at a given location from a given earthquake event is not a single deterministic number, but is in the form of a probability distribution with median and standard deviation. Where the ground motion is known exactly (i.e. at strong motion stations), the uncertainty (i.e. standard deviation) in the prediction is zero. However, for general locations uncertainty exists. It is strongly recommended that this uncertainty in PGA is considered by those tasked with liquefaction assessments. An simple example of how such uncertainty consideration can be performed was given.

6. References

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